Pascal's wager

Hacking (1972) distinguishes three separate logical arguments in the passage from *Pensées* discussing the wager. The discussion is based on a game-theoretic analysis, and can be understood by assuming the following payoff matrix:

	God exists	God does not exist
Wager for God	$u_1 = +\infty$	u_2 = finite
Wager against	$u_3 = -\infty$ or finite	u_4 = finite

The first argument Hacking (1972) describes he calls the "argument from dominance." In game theory, one strategy is said to dominate if its payoff is better than alternative strategies in at least one case, and is never less than the payoff for alternative strategies. For Pascal's wager, it is clear that, if God exists, $+\infty > u_3$, and Pascal goes on to argue that nothing is lost by wagering for God if God does not exist, and there are gains to reaped in this life, implying $u_2 \geq u_4$. For the environmental analogue, clearly $u_1 > -\infty$. However, for the argument from dominance to succeed, we must also show that $u_2 \geq u_4$. This latter condition is likely to be at least as contentious in the environmental sphere as it is in theological discussions, so an argument from dominance will not generally be persuasive in urging precautionary action.

The second logical argument is called the "argument from expectation" (Hacking 1972). Here Pascal develops an argument based on the probabilities associated with various outcomes in a manner akin to the modern concept of expected utility. Pascal at first assumes that there is an equal chance that God that exists or not. The assumption of equiprobability is reminiscent of the "principle of indifference" used in assigning probabilities to random events such as a coin toss or roll of dice (Jordan 2006, p.22). Pascal proceeds to consider the case where probability that God exists is assigned any non-zero value, p, which Hacking calls the "argument from dominating expectation." Based on this assumption the expected utility (EU) of each of the wagers can be calculated:

$$\text{EU(Wager for God)} = \quad p\cdot(+\infty) + (1 - p)\cdot(u_2) = +\infty$$

$$\text{EU(Wager against God)} = \quad p\cdot(u_3) + (1 - p)\cdot(u_4) = \text{finite}$$

Thus, a wager for God yields a higher (i.e., infinite) expected value. Pascal concludes one should wager for God, even if one thinks it is very unlikely that God exists.

An environmental analogue of Pascal's wager

These arguments can be adapted to an environmental analogue of Pascal's wager (Haller 2000). We assume the following payoff matrix:

	Catastrophe would occur	Catastrophe would not occur
Wager for catastrophe (precautionary action)	u_1 = finite	u_2 = finite
Wager against (no precautionary action)	$u_3 = -\infty$	u_4 = finite

We can calculate expected utilities in the environmental catastrophe wager as:

EU(Precautionary Action) = $p \cdot (u_1) + (1 - p) \cdot (u_2)$ = finite

EU(No Precautionary Action) = $p \cdot (-\infty) + (1 - p) \cdot (u_4) = -\infty$

Since any finite payoff is always better than $-\infty$, it follows by the argument of dominating expectation that we should take precautionary action. Note that this is true no matter what the relative values of u_2 and u_4 (unlike the argument from dominance), and even in cases where the probability of catastrophe (p) is very small.

If we relax the assumption of an infinite negative payoff if the catastrophe occurs, and make all the payoffs finite, the payoff matrix can be represented as:

	Catastrophe would occur	Catastrophe would not occur
Wager for catastrophe (precautionary action)	$u_1 = -k$	$u_2 = -k$
Wager against (no precautionary action)	$u_3 = -c$	$u_4 = b$

where $-k$ is the cost associated with precautionary action, $-c$ is the cost of the catastrophe, and b is the payoff if precautionary action is not taken and a catastrophe does not occur. Since precautionary action often translates into avoiding some risky but potentially beneficial alternative, b might be assumed to be positive. In accordance with the notion that $-c$ represents a catastrophic disutility, assume that $|c| >> |k|$ and $|c| >> |b|$. Let p represent the probability that catastrophe would occur in the absence of precautionary action. The expected utility of each (pure) strategy can be calculated as:

$$\text{EU(Precautionary Action)} = p\cdot(-k) + (1-p)\cdot(-k) = -k$$

$$\text{EU(No Precautionary Action)} = p\cdot(-c) + (1-p)\cdot(b) = b-(b+c)p$$

Unlike Pascal's wager, the expected utilities are always finite, and the expected utility of not taking precautionary action decreases linearly with p. There will always be some value of p (specifically, $p = (k+b)/(c+b)$), below which the expected value of not taking precautionary action will exceed the expected value of precaution. Presumably this will only be true when p is quite small; however, the value of p is no longer irrelevant when catastrophes have finite disutility.

LITERATURE CITED

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