Discounting over time in economic theory

Consider the problem of the optimal consumption of a natural resource. Let *s* represent the stock of resource (e.g., biomass of timber, abundance of fish, volume of ore, etc.), and let *c* represent consumption of the resource, which for simplicity, is assumed constant. The dynamics of the resource under exploitation can be represented as:

$$\frac{ds}{dt} = f(s) - c \qquad \textbf{[1]}$$

where *f*(*s*) is a function representing the natural dynamics of the resource stock. From a utilitarian perspective, we assume that the goal of resource utilization should be to maximize the utility derived from consumption. Representing the utility of present consumption by *u*(*c*) and discounting exponentially in the future, the objective is to find:

$$\max \int_0^\infty u(c) e^{-\delta t} dt \qquad \textbf{[2]}$$

where δ is the rate at which utility is discounted, also know as the pure rate of time preference (Heal 2007). Applied to individuals, δ reflects the preference for immediate rather than delayed enjoyment, which may be seen as a rational attitude toward an uncertain future (Viscusi 2007), or less flatteringly as impatience (Kysar 2007). Whatever the merits of a pure rate of time preference for individuals, its application in an intergenerational context is problematic (Cowen 2007, Heal 2007). It is not clear why the enjoyments of future generations should count for less simply because they occur in the future.

Discounting of utility is not the only motivation for discounting in economic analyses. Economists typically discount consumption based on the equation:

$$\rho = \delta + g\eta \qquad \textbf{[3]}$$

where ρ is the discount rate applied to future per-capita consumption, δ is the pure rate of time preference, *g* is the rate of growth of per capita income, and η is minus the elasticity of marginal utility with respect to consumption (Summers and Zeckhauser 2008). The rate at which consumption is discounted, ρ, is sometimes called the social discount rate (Heal 2007). The second term of the equation implies that, in a growing economy, even if no discount is applied for pure time preference, an increment of consumption will be valued less in the future because of its lesser marginal utility as income rises. In this equation, per capita income is used as a surrogate for per capita resource consumption, implying:

$$g = \frac{1}{N}\frac{dI}{dt} \propto \frac{1}{c}\frac{dc}{dt} \qquad [4]$$

$$\eta = -c\,\frac{\partial^2 u / \partial c^2}{\partial u / \partial c} \qquad [5]$$

where *I* is total income, *N* is the number of consumers, and the other variables retain their previously stated meanings.

Instead of the standard discounting procedure, one could apply a non-constant discount rate, particularly one that declines over time (Groom et al. 2005). The most widely suggested form of a declining discount rate is one where the discount is proportional to the logarithm of time. In that case, we could re-write equation [2] as follows:

$$\max \int_0^\infty u(c)e^{-\delta \log t}dt = \max \int_0^\infty u(c)t^{-\delta}dt \qquad [6]$$

This form of discounting is variously referred to as hyperbolic discounting (Ainslie and Haslam 1992), logarithmic discounting (Heal 1998) or gamma discounting (Weitzman 2001). Heal (1998, pp.62-63) justifies this form of discounting as an expression of the Weber-Fechner law, an empirical generalization that human response many stimuli (e.g., sound or light) is proportional to the logarithm of the stimulus intensity. Weitzman (2001) presents an entirely different justification, arguing that if there is substantial disagreement among individuals about the proper exponential discount rate to apply, the effective social discount rate will decline over time. Weitzman (2001) conducted a survey of over 2000 professional economists, asking them to provide their best estimate of the discount rate that should be applied in evaluating projects to mitigate global climate change. The distribution of estimates was well approximated by a gamma distribution, which fit a model with a declining discount rate (approximately 4% for time horizons less than 5 years, declining to near zero for horizons greater than 300 years).

LITERATURE CITED

Ainslie, G. and N. Haslam. 1992. Hyperbolic discounting. Pages 57-92 in G. Lowenstein and J. Elster, editors. *Choice Over Time.* Russell Sage Foundation, New York, NY, USA.

Cowen, T. 2007. Caring about the distant future: why it matters and what it means. *The University of Chicago Law Review* 74:5-40.

Groom, B., C. Hepburn, P. Koundouri and D. Pearce. 2005. Declining discount rates: the long and the short of it. *Environmental and Resource Economics* 32:445-493.

Heal, G. 1998. *Valuing the Future: Economic Theory and Sustainability.* Columbia Univ. Press, New York, NY, USA.

Heal, G. 2007. Discounting: a review of the basic economics. *The University of Chicago Law Review* 74:59-77.

Kysar, D. A. 2007. Discounting ... on stilts. *The University of Chicago Law Review* 74:119-138.

Summers, L. and R. Zeckhauser. 2008. Policymaking for posterity. *Journal of Risk and Uncertainty* 37:115-140.

Viscusi, W.K. 2007. Rational discounting for regulatory analysis. *The University of Chicago Law Review* 74:209-246.

Weitzman, M. L. 2001. Gamma discounting. *American Economic Review* 91:260-271.