

Appendix 1. Irrigation System Model - Description

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The model description below follows the ODD (Overview, Design concepts, Details) protocol for describing individual- and agent-based models (Grimm et al., 2006, 2010).

1. Purpose

The purpose of this model is to examine equity and efficiency in crop production across a system of irrigated farms, as a function of maintenance costs, assessed water fees, and the capacity of farmers to trade water rights among themselves.

2. Entities, state variables, and scales

This model consists of farm agents in a two-dimensional space, connected to an irrigation system. The irrigation system is a set of links (channel segments) connecting nodes, where farms are connected to the system at nodes (Figure 1).

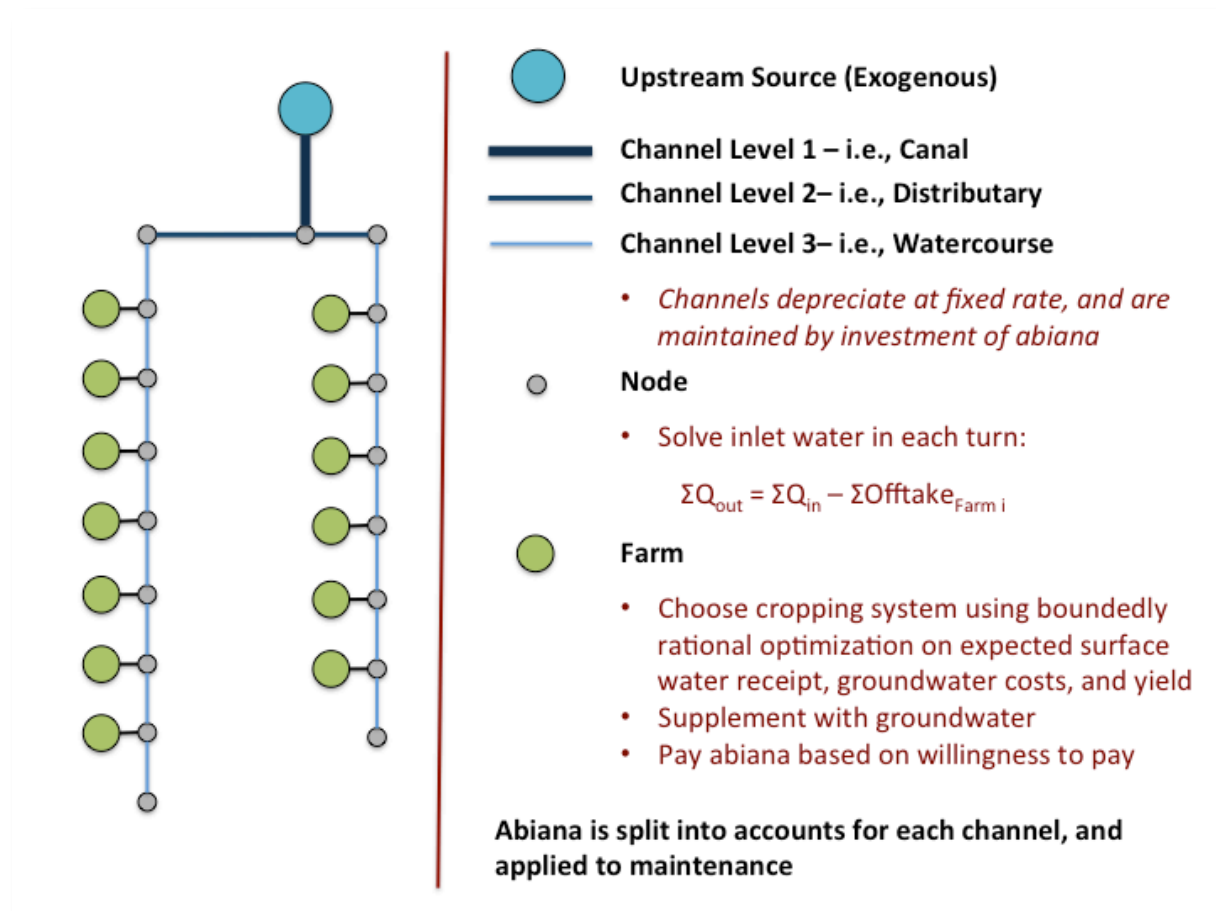


Figure 1 – Irrigation Model Overview

Farms have a unique ID and location in the 2-D space, and are described by the state variables:

- Size
- Wealth
- Water allocation (m^3/day)
- Current water receipt (m^3)
- Relative risk aversion coefficient
- Discount rate
- Memory of water receipt
- Land use portfolio (crop rotations allocated some fraction of overall farm water and farm land)

Irrigation channel segments have a unique ID and location in the 2-D space, and are described by:

- Design flow (m^3/day)
- Maintenance level
- Depreciation rate
- Irrigation level (canal, distributary, watercourse, etc.)
- Inlet node ID
- Outlet node ID
- Current water (m^3)

Irrigation nodes have a unique ID and location in the 2-D space, and are described by:

- Inlet link IDs
- Outlet link IDs
- Withdrawing farm IDs

The physical environment is described only by inlet water to the system, where the inlet represents any upstream portion of the broader irrigation system. Remaining variables in the modeled environment include the set of crops available to the farmers, the market prices (fixed and exogenous) that each crop earns, as well as the size of the water allocation increment (δ) and the number of increments δ that can be traded in a given decision time step.

Farms are organized in separate markets, with a market consisting of all farms along a non-branching series of channel segments. Farms located at branching nodes participate in the market upstream of them. Farmers within the same market can trade portions of their overall water allocation (up to a fixed number of increments δ) with each other.

Spatial scale in this model is arbitrary. There are two time scales of interest – i) the water turn time scale and ii) the farm decision time scale. The water turn time scale is the time required for one full set of ‘irrigation turns’; that is, the time across which each farmer can be expected to receive water, irrespective of the local rules for sequential access. Water distribution is determined simply by propagating water through the channel network and meeting farm water allocations until water is consumed, such that any actions to withdraw water (opening and closing gates) or timing of water consumption within the water turn time step is implicit in the farms’ water allocation and receipt. In our simulations the water turn time step is taken as 10 days. Depreciation of canal infrastructure also is updated at the water turn time scale.

The farm decision time scale represents the interval at which farmers revisit their plans for their land use, are assessed and pay water use fees (which are used at the same interval to maintain irrigation infrastructure), and have the opportunity to trade portions of their water allocation among other farms in the same market. In these simulations the farmer decision time step is taken as a crude one-year period – 36 water turn time steps, or 360 days. As a note, each of these separate decision processes (land use, water fees, and markets) could occur with different frequency, but in our simulations all occur with the same frequency.

3. Process overview and scheduling

The model solution scheme is as follows:

```

While t <= tmax
  For all canal links
    depreciateCanalInfrastructure
  If (mod(t, Δtdecision) = 0)
    For all farms
      updateLandUse
  For all canal links
    solveInletWater
  If (mod(t, Δtdecision) = 0)
    For all farms
      collectAbiana
  For all farms
    updateFarmerMemory
  If (mod(t, Δtdecision) = 0)

```

```

    For all canal links
        maintainCanalInfrastructure
    For all markets
        tradeWaterAllocation
End

```

Each of `depreciateCanalInfrastructure`, `updateLandUse`, `solveInletWater`, `collectAbiana`, `updateFarmerMemory`, `maintainCanalInfrastructure`, and `tradeWaterAllocation` are described below in *Submodels*.

4. Design concepts

Basic principles. The structure of this model is derived loosely from the functioning of large public irrigation systems in South Asia (e.g., Barker & Molle, 2004) with systems of water fees assessed for the maintenance of the system (e.g., Bell, Shah, & Ward, 2014) and some assignment of water allocation from which it is known there is deviation (trading, overage, theft, etc.) (Bandaragoda, 1998). It captures an irrigation context where the availability of water is a limiting factor in cropping decision-making, and where water availability shapes willingness to pay assessed fees, such as Pakistan (Bell et al., 2014). Farmers' use of their knowledge about water to make decisions is boundedly rational (Kahneman, 2003), captured through the use of a genetic algorithm (e.g., Manson & Evans, 2007; Manson, 2005) with a land-use portfolio acting as a gene (described in *Submodels* – `updateLandUse`), and a fitness function based on the farmers' expected utility (Feder, Just, & Zilberman, 1985) from marketing the crops grown under each portfolio.

Emergence. The key system-level outcomes of efficiency (yield, value, and diversity of crops produced) and equity (distribution of generated wealth across farms) emerge from farm decisions on how to use their land and whether to participate in the trading of water allocations.

Adaptation. Farms adapt their land use in successive decision time steps via the genetic algorithm described in `updateLandUse`, which searches for a land use portfolio with the highest expected utility, given current land use and the current memory of water receipt. Additionally, farms estimate their expected utility under different conditions of water allocation, and use this information to generate bid and offer prices of water allocation increments δ for participation in a water market; the market mechanism allows farms to adapt by moving toward water allocations that might benefit them further.

Objectives. Farm choice of land use, as well as participation in water allocation markets, is governed by the objective of maximizing expected utility (Feder et al., 1985).

Learning. Farms update their pool of candidate land-use portfolios in each successive iteration of the genetic algorithm.

Prediction. Farms predict expected utility for a given land-use portfolio by estimating future water receipt (based on a stored memory of previous water receipt) and from this, estimating yields using the FAO crop yield response to water model (Steduto, Hsiao, Fereres, & Raes, 2012), transformed into Jensen's sensitivity index (Kipkorir & Raes, 2002). This simple model (which breaks a crop's growth into phases with different sensitivities to water stress) is used as a representative understanding held by all farms (it is not learned, but rather is known) of how different crops will perform to a given water scenario.

Sensing. Farms observe and remember water reaching their farm. While they do not explicitly store memory for water receipt above or below them in the irrigation system, the routine for

evaluating potential water receipt under a change in water allocation of $n\delta$ searches the water memories of upstream and downstream farms in order to estimate actual water receipt under the change, so that some knowledge of neighboring farms' water receipt is implicit. Crop responses to water stress and crop prices are known and fixed. During the clearing of possible markets for water allocation, markets are solved using a knapsack algorithm (Strandmark, 2009) in which the ability for the sellers with the highest offer price to access the buyer with the highest bid is implicit.

Interaction. Farms interact directly only through participation in the trading of water allocation, in `tradeWaterAllocation`. Farms interact indirectly with all farms downstream of them through any trade in water allocation, by changing the potential amount of water that will reach downstream farms.

Stochasticity. Stochasticity is introduced across many parts of the model. Specifically, it appears:

- In model setup, to draw farm-level parameters for size, risk coefficient, discount rate, and the number of years used in estimating expected utility
- In model setup, to randomly select the fidelity with which a farm uses their memory in decision making (i.e., remembering the past year as 36 distinct water turns, 4 distinct seasons, 1 average year, etc.)
- In the main algorithm, to randomize the order of agents in each new decision time step
- In the main algorithm, to estimate inlet water in each water turn time step
- In `maintainCanalInfrastructure`, to randomize the order through which channel links are maintained, if this option is selected (alternative is to order from worst to best condition)
- In the genetic algorithm within `updateLandUse`, to generate new candidate land use portfolios, select points for crossover and mutation, and as part of the selection procedure (whether probabilistic or tournament) for inclusion in the next generation
- In the evaluation of expected utility, in the selection of past cycles of water memory to be used in estimating future water receipt

Collectives. Farms interact via markets, with a market composed of all farms connected to nodes along a non-branched segment of irrigation channel, inclusive of farms connected to the downstream node at which branching occurs. Farms in the same market are able to trade portions of their water allocations with each other in each decision time step.

Observation. In our experiments, outcomes of i) the average value-of-production (VOP, averaged over the duration of a simulation and across the landscape), ii) crop income diversity (measured via the Shannon index of crop revenues over the simulation, across the landscape), iii) farm wealth distribution (measured via a Gini coefficient), and iv) farm water allocation distribution (measured via a Gini coefficient) are used as key outcomes. All farm-level attributes as well as farm-level crop incomes are returned from the simulation.

5. Initialization

At initialization, farms have no previous memory of water receipt, or candidate land use portfolios for consideration. At $\Delta t_w = 0$ (i.e., the water time step is 0), the random seed for the simulation is set, the landscape is initialized, and the simulation is run for a period of 10 full decision time steps (in our simulations, 360 Δt_w) without the farms taking any action, in order

to accumulate a memory of water receipt (i.e., a spin-up period). The first decision timestep Δt_D thus occurs at $\Delta t_w = 361$.

Table 1 summarizes model parameterization for our chosen set of experiments. Genetic algorithm parameterization is based on that of Manson (2005). True costs for maintenance of irrigation systems in South Asia are not well known, because at best revenues and spending are recorded, rather than indication of true maintenance and repair needs (Malik, Prathapar, & Marwah, 2014); in our experiments we choose local and global irrigation maintenance costs to cover a range of conditions (from insignificant to limiting cost levels). Abiana levels in these experiments are chosen as well to span a range of conditions, but notably this range begins at a level above current water use fee assessment for Pakistan (as a representative of large-scale public irrigation in Asia) and stops well below farmers' measured willingness to pay for reliable canal water (Bell et al., 2014).

	Parameter	Value(s)	Unit	Notes
System parameters	Water turn time step (Δt_w)	10	days	
	Decision time step (Δt_D)	360	days	
	Number of farms	24	farms	
Irrigation system parameters	Share of water use fees prioritized for inlet maintenance	0.7		
	Share of water use fees prioritized for canal maintenance	0.2		
	Share of water use fees prioritized for lower-level maintenance	0.1		
	Maintenance scheduling	1		0 is random; 1 is ordered from lowest maintenance to highest
	Maintenance increment	0.01		Incremental improvement before moving to next channel segment
	Depreciation Rate	0.0002		Rate of decay in maintenance level of channel segments per water turn time step
	Inlet depreciation rate	0.002		Rate of decay in maintenance level of inlet water maintenance per water turn time step
	Initial Inlet maintenance	0.3		
	Initial Irrigation channel maintenance	1		
	δ	0.03		Increment of water allocation used for trading in water markets (Calculated after total size of farms is generated, providing 5mm for every hectare of land in system. This value of 5mm is the reference evapotranspiration rate used by the FAO crop water model (FAO, 1998))
Farm parameters	Inlet Design Flow	5	mm/ha/d	
	Farm risk coefficient, μ	0.6		Constant relative risk aversion coefficient
	Farm risk coefficient, σ	0.2		
	Farm discount rate, μ	0.1		

	Farm discount rate, σ	0.02		
	Farm size, μ	15	hectares	Minimum farm size is truncated at 2
	Farm size, σ	10	hectares	
	Farm years ahead, μ	20	years	Number of years ahead used in estimating expected utility
	Farm years ahead, σ	3	years	Number of years ahead used in estimating expected utility
Genetic algorithm parameters	Minimum plot size	0.25	hectares	Minimum size of a plot within a land use portfolio in the genetic algorithm
	Max spacing	8	Δt_w	Maximum fallow period between cropping cycles in crop rotation
	Zero turn	50	Δt_w	Probability of adding another crop-fallow cycle scales from 1 down to 0 at 'Zero turn'
	Population size	50	portfolios	
	Number of Generations	10		Number of generations per decision time step
	Number of δ Generations	1		Number of generations to use when estimating expected utility of current water allocation + $n\delta$
	Probability of crossover	0.9		Parameter settings from Manson (2005)
	Probability of mutation	0.01		Parameter settings from Manson (2005)
	Probability of direct reproduction	0.09		Parameter settings from Manson (2005)
	Selection method	3		1 - Probabilistic selection; 2 - Tournament selection; 3 - Elite tournament selection
	Tolerance for early exit	0.1		Fractional change in expected utility below which algorithm is considered to have settled and can exit early
	Generations for tolerance	5		Number of consecutive rounds for which change in expected utility must be below tolerance for early exit
	Years data	5	years	Years of water memory data used for estimating future water data
	Sweep parameters	$n\delta$	{0, 2, 5}	
Water use fee		{1000, 2000, 5000, 10000}		Rs. /ha / Δt_D
Inlet maintenance cost (per farm in simulation)		{200000, 500000, 1000000}		Rs. / (0.01 change)
Irrigation channel maintenance		{200000, 500000, 1000000}		Rs. / (0.01 change) / unit length channel
Number of channels		{1, 2, 3}		number of separate channels across which the farms are allocated

6. Input data

The model draws external data on crop yields, costs, and prices. Specifically, for any crop (or particular sequence of crops) to be considered within the genetic algorithm, data must be provided for each water turn time step for i) crop phase, ii) the crop coefficient K_c and iii) the

yield response factor K_y (Steduto et al., 2012). Additionally, overall cost data must be provided for this duration, broken apart into i) startup fixed costs (i.e., any costs associated with growing this crop or sequence for the first time), ii) per-season fixed costs (i.e., any fixed costs associated with each new application of the crop or crop sequence), and iii) per-area variable costs (i.e., any variable costs associated with applying this crop or crop sequence to a unit area). Finally, the nominal expected yield (without any water stress) should be provided, as well as a per-unit price. Sample crop data sheets, in the correct format to be read by the model, are included with this protocol.

7. Submodels

This model includes submodels `depreciateCanalInfrastructure`, `updateLandUse`, `solveInletWater`, `collectAbiana`, `updateFarmerMemory`, and `tradeWaterAllocation`.

7.1 `depreciateCanalInfrastructure`

In this submodel, the maintenance levels m_i for all irrigation channel segments i , as well as the maintenance level for the inlet (which represents all irrigation infrastructure upstream of the modeled system, are depreciated by $m_i = m_i * (1 - d_i)$, where d_i is the appropriate depreciation rate (either for channel segments or for the inlet).

7.2 `updateLandUse`

This submodel integrates several different routines to capture the actions in the farm decision time step. An overview of the submodel (in pseudocode) is as follows:

```

For each farm (in random order)
  • Calculate yields for any crops harvested over the previous decision time step
  • Estimate best new land-use portfolio using genetic algorithm
  • Decide whether to switch to best new portfolio or stick with current portfolio
  • Incur any costs from upcoming decision time step
  • Estimate WTP and WTA for participation in water allocation market in this time step using genetic algorithm
End

```

7.2.1 Yield calculation

The same routine is used both for calculating yields over the previous period as well as for estimating possible yields within the genetic algorithm, and employs the Jensen crop water production function (Kipkorir & Raes, 2002):

$$\frac{Y_a}{Y_m} = \prod_{i=1}^n \left(\frac{ET_a}{ET_{c,i}} \right)^{\lambda_i}$$

where Y_a is the actual yield, Y_m is the maximum yield with no water stress, ET_a is the actual evapotranspiration, and $ET_{c,i}$ is the evapotranspirative demand in phase i (over n total phases). The evapotranspirative demand for a phase is estimated as $ET_0 * K_{c,i}$, where ET_0 is the standard reference evapotranspiration of 5mm/day (FAO, 1998) and $K_{c,i}$ is the crop coefficient for phase i . The exponent λ_i is converted from the yield response factor K_y by the polynomial method outlined in Kipkorir and Raes (2002):

$$\lambda_i = 0.2757K_{y,i}^3 - 0.1351K_{y,i}^2 - 0.0761K_{y,i} - 0.0107$$

7.2.2 Estimating best land-use via genetic algorithm

Farmer land use is described by a portfolio of crop rotations, each allocated a fraction of the farm land and a fraction of the overall water allocation (Table 2, for an example).

Area	Water Fraction	Rotation
0.6 ha	0.6	Rice, 3 time step break, Wheat, 2 time step break, Corn, 5 time step break
0.4 ha	0.25	Sugarcane, 2 time step break
1.2 ha	0.15	Onions, 2 time step break, Onions, 6 time step break

Table 2 – Example Land-use portfolio for a 2.2 ha farm

The genetic algorithm employed in this model uses the portfolio as a gene, and individual rotations (together with their area and fractional water allocation) as the trait. The algorithm follows the same design used by Manson (2005) and Manson & Evans (2007), in which member genes are selected to reproduce either by i) probabilistic, ii) tournament, or iii) elite tournament selection; and in which reproduction follows either i) crossover between two parents, ii) mutation of a single parent, or iii) direct reproduction without change. What makes any genetic algorithm unique is the interpretation of crossover, mutation, and the appropriate fitness function, which we describe next.

Crossover is performed as a simple shuffling of crop rotations – the set of all crop rotations belonging to the two parent portfolios is pooled, and then each rotation is randomly allocated to one of the two child portfolios. Areas for each rotation are re-scaled to sum up to the total actual farm size, and water allocations are rescaled to sum to 1.

Mutation is allowed to occur in any part of the portfolio. Specifically, one rotation is selected randomly, and within this rotation a single point mutation type is drawn randomly (with equal probability for each): i) area mutation, ii) water mutation, or iii) crop mutation. In the case of an area mutation, the fraction of the farm's land allocated to that rotation is randomly mutated, and all land areas are then re-scaled to sum up to the size of the farm. The minimum fraction of a farm that a rotation can occupy is constrained, so that in some cases this rescaling process must be iterative, setting areas that are too small to the minimum size and rescaling. In the case of water fraction, mutation proceeds in a similar manner – randomly mutating the fraction of water allocation to the current rotation, then rescaling all other fractions to sum to 1. In this case, there is no minimum water fraction to allocate to a rotation, so that this is never an iterative process. Finally, the case of mutating the crop rotation is a step-wise set of decisions. First, one crop-fallow pair in the rotation is selected randomly. Next, it is selected whether to delete this crop-fallow pair, or to add an additional crop-fallow pair, with equal probability. In the case of adding a crop-fallow pair, it is selected whether to add the pair before or after the currently selected pair, and then the actual crop and fallow period are drawn randomly.

The fitness function for the genetic algorithm is expected utility for the portfolio, given known water history, calculated as:

$$E(U) = \frac{1}{n} \cdot \sum_n \frac{\left[\sum_{crops\ i} [(P_i Y_i)(1+d)^{t-t_h} - \sum_{costs\ j} C_{i,j}(1+d)^{t-t_j}] \Big|_{water\ history\ k} \right]^{(1-r)}}{1-r}$$

where P_i is the price for crop i , Y_i is the yield of crop i , d is the discount rate, t_h is the time of harvest, $C_{i,j}$ is the cost of type j (fixed or variable cost) incurred at time t_j , r is the risk coefficient, and n is the number of different, equally likely water histories.

The idea of ‘equally likely water histories’ is perhaps easiest explained by example. Consider a portfolio in which the longest rotation is 4 decision time steps (we write ‘season’ for simplicity) in length, while the second rotation is 3 seasons and the last is 1 season. If we have 5 seasons of data, then there are 2 unique histories to evaluate the 4-season rotation (starting in season 1 and starting in season 2), 3 unique histories to evaluate the 3-season rotation, and 5 unique histories to evaluate the 1-season rotation. However, if we restrict ourselves to water patterns that have some overlap (i.e., there is no overlap between the 3-season history that starts in season 1 and the 1-season histories that start in seasons 4 or 5), then we have a slightly reduced set of possibilities to consider. We are interested in the 3 different ways the 1-season rotation could occur within the 3-season rotation. In turn, the 3-season rotation can occur in 2 different ways within each of the 4-season rotation, which in turn can occur 2 different ways within the 5 years of data. Our number of unique histories is thus $n = 3 \times 2 \times 2$ (as opposed to $n = 5 \times 3 \times 2$). In the event that the longest rotation in a portfolio is longer than the available data, existing cycles are repeated randomly until the memory data is as long as the longest rotation.

The same method is used to evaluate utility remaining the currently active portfolio by evaluating only crop plantings and harvests that have yet to occur.

7.3 solveInletWater

This submodel estimates incoming water for the current water turn time step and calculates its propagation and withdrawal through the system.

First, incoming water is estimated by:

$$Q_{inlet} = [(1 - M) \cdot rand() + M] \cdot (Q_{Design})$$

where M is the level of maintenance of the inlet. In this way, a perfectly maintained inlet will provide water at the design flow rate, while a poorly maintained inlet will have a very random stream. Alternatively, a schedule of water data (such as might be available from an irrigation department) could be used, in order to capture events like planned shutdowns, etc., though this is not currently undertaken with this model.

This submodel operates by propagating available water through the irrigation channel segments to nodes. The fraction of water lost by each channel segment i is equal to $(1 - M_i)$, where M_i is the maintenance level for that channel segment. Water reaching a node is given first to any farms connected to that node, up to their water allocation or the amount of water remaining in the channel. If there is water remaining at a node after giving to farms, it is allocated proportionally among any outlet links from that node, based on the cumulative water demand of each outlet link (the sum of all water allocations of farms downstream along that outlet channel segment, not considering leakage through low maintenance). This process repeats along each channel segment and node until terminal nodes are reached.

Water remaining at terminal nodes is considered drainage and is set to 0.

7.4 collectAbiana

Farms choose to pay water use fees (*abiana*, in the case of Pakistan) in this model according to the following schedule:

$$Water\ Use\ Fee\ Paid = (Farm\ Size) \cdot \min\left(Assessed\ Fee, 23000 \cdot \frac{Water\ Received}{Water\ Allocated} \right)$$

where the value 23000 represents the cumulative willingness to pay of approximately 23000 Pakistan Rupees per hectare for a reliable water supply measured by Bell et al. (2014). This simple model scales water payments from 0 (when no water is received) up to a maximum of 23000 Rs or the assessed water use fee per hectare (when all water allocated is received).

Fees received are allocated to separate channel accounts, with farms contributing to channels through which they receive water only, and with proportional allocation of the fees across the inlet and other channels fixed by the irrigation system parameters described in Table 1.

7.5 *updateFarmerMemory*

In this submodel, the array of water memory held by the farm is updated to integrate the previous water turn timestep.

7.6 *maintainCanalInfrastructure*

This submodel applies collected water use fees to the maintenance of modeled irrigation channel infrastructure, as well as to the inlet water (which represents all irrigation infrastructure upstream of the modeled section, exogenous to the current model).

Available funds are allocated to separate accounts for each channel during the water use fee collection (Submodel 7.4 – collectAbiana). For each of these channels, funds are applied to maintain the irrigation channel segments, either 1) in random order, or 2) in order from lowest maintenance level to highest, depending on parameter settings. In a given segment, the maintenance level is increased only by the increment specified in the irrigation parameter settings before moving on to another segment. If there are funds remaining once all segments in the channel that require maintenance have been raised by this increment, this process is repeated until all funds for this channel have been used or all segments are fully maintained.

After completing maintenance on the modeled irrigation channels, all unused funds are added to the account for maintaining the inlet water. The total funds available are then applied, as necessary, to raising the maintenance level of the inlet water – which in practice would include the maintenance, repair or even new development of irrigation channels, barrages, dams, pumps, etc.

7.7 *tradeWaterAllocation*

This submodel solves a market for the 'lumpy' commodity that water is in the current context. In agricultural systems, the marginal value of additional water supply may vary unevenly. For instance, a farm with more than enough water to grow wheat but not enough water to grow sugarcane might have a low marginal value for a small additional amount of water (since they can not use it to their advantage) but a high marginal value for a larger amount of water (if it enables them to transition to sugarcane). At the same time, they may be quite interested in selling water. This can be a difficult market problem to resolve, as agents have the potential to participate in the market in very different ways, depending on what other offers are available.

If the willingness of each farmer to participate in a market can be evaluated at several different points, then the overall market can be solved using solvers for the 'knapsack problem' (Strandmark, 2009), which find the most valued set of elements that add to a given weight constraint.

Specifically, this submodel receives a list of all bids that farms in a market are willing to make on increments of δ through $n\delta$ of water allocation, and a separate list of prices at which the same farms would be willing to sell increments of δ through $n\delta$ of water allocation. These bids and prices are evaluated by estimating the expected utility of the farmers' current allocations (and actual receipt) modified by adding δ through $n\delta$ (to calculate the bids for purchasing) and by subtracting δ through $n\delta$ (to calculate the prices for selling). Note that a change in water allocation of δ is not the same as a change in water receipt of δ – the submodel looks at the actual water receipt histories of neighboring farms to determine what change in receipt would actually have occurred from a change in allocation of δ . By estimating the change in expected utility under a change in allocation of δ , the willingness to pay (or willingness to accept, in the case of a sale) for that change δ can be estimated as:

$$WTP_{\delta} (WTA_{\delta}) = \left(U_{+s1-\delta} (1-r) \right)^{1/r} - \left(U(1-r) \right)^{1/r}$$

In a simulation where the number of allowed increments is n , each farm will have WTP and WTA estimates for 1 through n increments δ (with implied *marginal* WTP and WTA values $MWTP = WTP/n$). The list of all bids across all farms in the market is ordered from greatest to least, and the knapsack problem is solved for each one in turn, until there are no more possible transactions. A transaction is possible if there is a set of increments for sale such that the total price for the increments offered is below the willingness to pay for the total set of increments (e.g., a bid of 18 for 4δ could be met by 3δ offered for 12 and δ from another farm offered for 5) – this is the solution to the knapsack problem. The final price paid is calculated separately for each selling farm as the mean of the WTP of the buyer and the WTA of the seller. Once a farm has participated in a transaction, either as a buyer or a seller, they leave the market (for this timestep) and do not participate in further transactions until the next decision timestep.

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