## Appendix A. Appendix

### *Basic model*

Basic model. Here we briefly describe the basic model presented by Muneepeerakul and Anderies (2017). The model shows dynamic behaviour of three principal variables, namely, the state of the public infrastructure, $I_{HM}$, resource level, $R$, and the fraction of time user makes use of infrastructure, $U$, through Eqs. A.1. A.4. A.5. The schematic diagram of this system of equations is shown in Fig. A1.

In this context, $I_{HM}$ depends on PIPs in term of maintenance cost and has a positive relationship with the capacity of users to create resource flows. Eq. A.1 illustrates the dynamics of $I_{HM}$ as follows:

$$\frac{dI_{\mathrm{HM}}}{dt} = M(...) - \delta H(I_{\mathrm{HM}}) \tag{A.1}$$

where $\delta$ is the infrastructure's depreciation rate and $H(I_{HM})$ states functional relationship of public infrastructure and productivity of each RU. According to Muneepeerakul and Anderies (2017), many shared infrastructures can be modeled by threshold functions. Given that $H(I_{HM})$ shows threshold behavior, they used a piecewise linear function to capture such behavior through Eq. A.2.

$$H(I_{HM}) = \begin{cases} h, & I_{HM} > I_m \\ h\left(\dfrac{I_{HM} - I_0}{I_m - I_0}\right), & I_0 \leq I_{HM} \leq I_m \\ 0, & I_{HM} < I_0 \end{cases} \tag{A.2}$$

where $h$ represents the maximum amount of harvest by each user under no restriction and $I_0$ and $I_m$ are the lower bound and upper bound thresholds of $I_{HM}$, respectively. Also, $M(\ldots)$ is the maintenance function (Eq. A.3) and depends on the social structure of the system.

$$M(...) = \mu y p C U N R H(I_{\mathrm{HM}}) \tag{A.3}$$

In Eq. A.3, given the number of users $N$, $RUNH(I_{HM})$ is the total harvest from the natural infrastructure. The RUs sell total harvest at price $p$ to generate revenue. Subsequently, they assign a proportion $C$ of revenue to PIP's for their contribution. Meanwhile, the PIP's spend proportion $y$ of $C$ on maintaining public infrastructure through the maintenance function $M(\ldots)$. Also, $\mu$ is the maintenance effectiveness of PIP's investment.

The second variable is the resource level, $R$. They assumed the dynamics of resource to be

$$\frac{dR}{dt} = G(R) - UNRH(I_{\mathrm{HM}}) \tag{A.4}$$

Natural infrastructure is assumed to invoke the conservation law comprising of regenerating capacity ($G(R) = g - dR$) and total unit of harvest, $RUNH(I_{HM})$. The definition presented for $G$ is the simplest model for natural infrastructure where $g$ and $d$ are the natural replenishment and the loss rates, respectively.

The strategic behavior of the resource users (RUs) is captured by employing a replicator equation. Indeed, replicator dynamics provide modelers with simple, realistic social mechanism where agents follow and replicate better-off strategies. The two possible strategies considered for RUs are staying inside the system with the associated payoff of $\pi_U = (1-C)pRH(I_{HM})$ or leaving the system with the payoff of w=w. According to the replicator equation:

$$\frac{dU}{dt} = rU(1-U)(\pi_U - w) \tag{A.5}$$

The replicator equation represents the fraction of time that RUs assign to working inside system given $C$ and $y$. Like RUs, there is also two alternatives for PIPs, working inside the system or working for another CIS which leads to system failure. Meanwhile, $C$ and $y$ characterize the strategy or policy of PIPs. The PIPs will participate in this coupled system only when $\pi_p = (1-y)pCRUNH(I_{HM}) \geq w_P$. In other words, the PIPs maintain the system when they are better-off than working outside. This condition is termed the PIP Participation Constraint (PPC).

Based on the system of three differential equations (Eqs. A.1. A.4. A.5), the sustainable equilibria, i.e., long-term system outcomes that satisfy the stability condition and PPC, can be expressed as follows:

$$\begin{aligned} i^*_{HM} &= \frac{yCU^*NR^*}{g} H(I^*_{HM}) \\ R^* &= \frac{g}{d}\left(\frac{i^*_{HM}}{yC}\right) \\ U^* &= \frac{1-C}{yC}\phi_1 i^*_{HM} \end{aligned} \tag{A.6}$$

where $i^*_{HM} = \frac{I^*_{HM}\delta}{\mu pg}$ (indicates dimensionless) and $\phi_1 = \frac{pg}{wN}$, a dimensionless group representing the relative lucrativeness of the system, namely the ratio of potential income – with the entire resource flow turned into income – relative to outside wage. The results reported in this study are based on the following parameter values: $h = 0.0005$; $\delta = 0.1$; $I_0 = 0.3$; $I_m = 3$; $g = 100$; $d = 0.02$; $N = 1000$; $r = 0.15$; $p = 10$; $w = 1$; $w_p = 100$; and $= 0.001$.

### *Simplifying Eq. 1 through timescale separation argument*

We first used the timescale separation argument to simplify the model: the dynamics of $R$ and $U$ are significantly faster than that of $I_{\mathrm{HM}}$ as establishment and development of human-made infrastructures is a time-consuming process. This argument, namely ($dR/dt = dU/dt = 0$), leads to

$$URNH(I_{\mathrm{HM}}) = g - \frac{lw}{(1-C)pH(I_{\mathrm{HM}})}$$

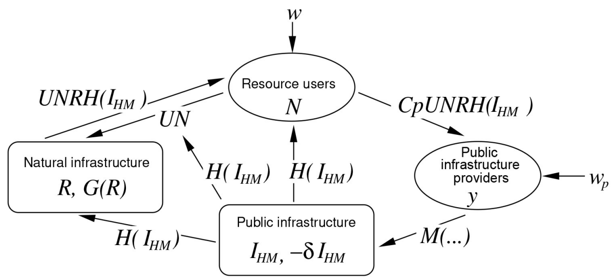


**Figure A1.** Schematic diagram of the dynamical system model. Taken from Muneepeerakul and Anderies (2017).

Applying such equality, the payoff associated with the PIPs, $\pi_\mathrm{P} = (1 - y)pCUNRH(I_\mathrm{HM})$, can be written as:

$$\pi_\mathrm{P} = (1-y)pC\left(g - \frac{lw}{(1-C)pH(I_\mathrm{HM})}\right) \tag{A.7}$$

Given the same argument, we derived $U$ as follows:

$$U = \frac{(1-C)p}{Nw}\left(g - \frac{lw}{(1-C)pH(I_\mathrm{HM})}\right) \tag{A.8}$$

***Non-dimensionlization***

Using some dimensionless groups, we rewrote the payoff associated with the PIPs as follows:

$$\pi_\mathrm{P} = (1-y)Cpg\left(1 - \frac{\rho}{(1-C)\phi_1 H(x)}\right) \tag{A.9}$$

where, $x = \frac{I_\mathrm{HM}}{I_\mathrm{m}}$ is the rescaled $I_\mathrm{HM}$. Also, $\phi_1 = \frac{pg}{wN}$ is defined as the potential income — with the entire resource flow turned into income — relative to outside wage and $\rho = \frac{l}{hN}$ indicates the relative natural loss rate of the resource compared to the maximum harvest rate. Accordingly, we can write the resilience-based objective function as follows:

$$(x - x_c)(\frac{\pi_\mathrm{P}}{w_\mathrm{P}} - 1) = (x - x_c)\left((1-y)C\phi_2\left(1 - \frac{\rho}{(1-C)\phi_1 H(x)}\right) - 1\right) \tag{A.10}$$

where $\phi_2 = \frac{pg}{w_\mathrm{P}}$ is a dimensionless group indicating potential income relative to PIP's alternative opportunities.

Likewise, we used the dimensionless groups to rewrite the Eq. A.8 as follows:

$$U = (1 - C)\phi_1 \left(1 - \frac{\rho}{(1 - C)\phi_1 H(x)}\right) \tag{A.11}$$

Using the above equation, we rewrote the performance-based objective function as:

$$U + \frac{\pi_P}{Nw} = (1 - yC)\phi_1 \left(1 - \frac{\rho}{(1 - C)\phi_1 H(x)}\right) \tag{A.12}$$

We also rewrote the Eq. 3 in dimensionless form (Eq. 4) in the main text using the above dimensionless groups. In this equation, $\theta_{\mathrm{m}} = \frac{\delta I_{\mathrm{m}}}{\mu p g}$ is the dimensionless group indicating decay rate of infrastructure at $I_{\mathrm{m}}$ relative to maximum maintenance.

### *Deriving $\hat{y}_{\mathbf{R}}$, $\hat{y}_{\mathbf{P-PIP}}$ and $\hat{y}_{\mathbf{P-SYS}}$*

In order to derive the governance policies that maximize the objective functions associated with the resilience and the performances of the system represented by Eq. 1, we applied the HJB approach (Eqs.8, 9 and 10).

We then evaluated the first-order condition by taking the derivative of the terms inside the sqaure brackets on the right-hand side of the HJB equation (Eq. 8) with respect to the control variable, $y$ (i.e. the investment policy). The derivative,

$$\left(1 - \frac{\rho}{(1 - C)\phi_1 H(x)}\right)\left[\frac{dV}{dx}\frac{C}{\theta_{\mathrm{m}}} - C\phi_2(x - x_c)\right],$$

reveals an on-and-off form of the policy associated with the resilience of the system: when the above expression is positive, we should invest all in the infrastructure maintenance, $y = 1$; otherwise, we should not invest at all, $y = 0$.

Likewise, evaluating the first-order condition for the Eqs. 9 and 10,

$$\left(1 - \frac{\rho}{(1 - C)\phi_1 H(x)}\right)\left[\frac{dV}{dx}\frac{C}{\theta_{\mathrm{m}}} - C\phi_1\right],$$

$$\left(1 - \frac{\rho}{(1 - C)\phi_1 H(x)}\right)\left[\frac{dV}{dx}\frac{C}{\theta_{\mathrm{m}}} - C\phi_2\right],$$

also result in on-and-off policies in case of the performance priority.

In this study, the dimensionless groups associated with the base line scenario (corresponding to $g = 100, w = 1$) are as follows: $\phi_1 = 1, \phi_2 = 10, \rho = 0.2, \theta_{\mathrm{m}} = 0.06$. For the purpose of the sensitivity analysis, we analyzed the model for different values of replenishing rates of resource, $g = \{90, 100, 110\}$, and per capita wage from working outside, $w = \{0.9, 1, 1.1\}$.